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Counting dyons in $\mathcal{N} = 8$ string theory

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Abstract: A recently discovered relation between 4D and 5D black holes is used to derive exact (weighted) BPS black hole degeneracies for 4D $\mathcal{N} = 8$ string theory from the exactly known 5D degeneracies. A direct 4D microscopic derivation in terms of weighted 4D D-brane bound state degeneracies is sketched and found to agree.

Keywords: Black Holes in String Theory, D-branes.

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Contents

JHEP06(2006)037

1. Introduction

In this paper, we deduce an exact formula for the modified elliptic genus of string theory in four dimensions with $\mathcal{N} = 8$ supersymmetry. The modified elliptic genus, as we review below, provides a weighted count of BPS states of $\mathcal{N} = 8$ string theory. We derive a formula for it using a recently proposed exact relation [1] between 4D and 5D BPS degeneracies, together with the known degeneracies [2] in 5D. In addition we sketch a direct microscopic counting of D0-D2-D4 bound states which gives the same result. Our hope is that this example will provide a useful laboratory for testing the string theory relations recently proposed in e.g.[3].

Some years ago an explicit formula for the elliptic genus for BPS states in 4D $\mathcal{N} = 4$ theories was presciently conjectured [4]. This formula was recently derived using the 4D-5D connection in [5]. The present work is an extension of [5] to 4D $\mathcal{N} = 8$ theories. Previous work in this direction includes [6 – 8].

In the next section 2 we review the 5D index defined and computed in [2]. In section 3 we use the 4D-5D connection to derive the 4D index. In section 4 we sketch how this expression should follow (for one element of the U-duality class of black holes) from a microscopic analysis.

2. Review of the 5D modified eliptic genus

In this section, we want to summarize the work of reference [2] on counting the microstates of 1/8 BPS black holes in five dimensions. These can be realized in string theory as the usual D1-D5-momentum system of type IIB on $T^4 \times S^1$, with Q_1 D1-branes, Q_5 D5-branes and integral S^1 momentum n. The reason that microstate counting of this system is more difficult than for $K3$ compactification is because the usual supersymmetric index that counts these microstates, the orbifold elliptic genus of $Hilb^k(K3)$ with $k = Q_1Q_5$, vanishes when $K3$ is replaced with T^4. In [2], this difficulty was overcome by defining (and then computing) a new supersymmetric index $\mathcal{E}_2$, closely related with the elliptic genus, which is nonvanishing for T^4. We will refer to this new supersymmetric index as the modified elliptic genus of $Hilb^k(T^4)$. It is defined to be

$$\mathcal{E}_2^{(k)} = Tr\left[(-1)^{2J_L^3-2J_R^3} 2(J_R^3)^2 q^{L_0}\overline{q}^{\overline{L}_0} y^{2J_L^3}\right] \tag{2.1}$$

where the trace is over states of the sigma model with target space $Hilb^k(T^4)$.[1] Here J_L^3 and J_R^3 are the left and right half-integral $U(1)$ charges of the CFT, and they are identified with generators of SO(4) rotations of the transverse R^4. The S^1 momentum is $n = L_0 - \overline{L}_0$. The usual elliptic genus is given by the same formula but without the $2(J_R^3)^2$ factor; it is these two insertions of J_R^3 that make $\mathcal{E}_2$ nonvanishing for T^4.

As for $K3$, here it is convenient to define a generating function for the modified elliptic genus:

$$\mathcal{E}_2 = \sum_{k\geq 1} p^k \mathcal{E}_2^{(k)} \tag{2.2}$$

In [2], this was shown to be given by the following sum

$$\mathcal{E}_2(p,q,y) = \sum_{s,k,n,\ell} s(p^k q^n y^\ell)^s \widehat{c}(nk,\ell) \tag{2.3}$$

with the sum running over $s, k \geq 1$, $n \geq 0$, $\ell \in \mathbb{Z}$. Note that the $\overline{q}$ dependence has dropped out — only the $\overline{L}_0 = 0$ states contribute to the modified elliptic genus. Of course, the index must have this property in order to count BPS states, since the BPS condition is equivalent to requiring $\overline{L}_0 = 0$.

It was furthermore shown in [2] that the integers $\widehat{c}(nm,\ell)$ are the coefficients in the following Fourier expansion

$$Z(q,y) \equiv -\eta(q)^{-6}\vartheta_1(y|q)^2 = \sum_{n,\ell} \widehat{c}(n,\ell) q^n y^\ell \tag{2.4}$$

where $\eta(q)$ is the usual Dedekind eta function, and $\vartheta_1(y|q)$ is defined by the product formula

$$\vartheta_1(y|q) = i(y^{1/2} - y^{-1/2})q^{1/8}\prod_{n=1}^{\infty}(1-q^n)(1-yq^n)(1-y^{-1}q^n) \tag{2.5}$$

Finally, it was observed in [2] that $\widehat{c}(n,\ell)$ actually only depends on a single combination of parameters $4n - \ell^2$:

$$\widehat{c}(n,\ell) = \widehat{c}(4n-\ell^2) \tag{2.6}$$

Using (2.6) in (2.3) yields

$$\mathcal{E}_2(p,q,y) = \sum_{s,k,n,\ell} s(p^k q^n y^\ell)^s \widehat{c}(4nk-\ell^2)\,. \tag{2.7}$$

When (k,n,ℓ) are coprime, $\widehat{c}(4nk-\ell^2)$ counts BPS black holes with $k = Q_1Q_5$, S^1 momentum n and spin $J_L^3 = \frac{\ell}{2}$, multiplied by an overall $(-)^\ell$ and summed over J_R^3 weighted by $2(J_R^3)^2(-)^{2J_R^3}$:

$$\widehat{c}(4nk-\ell^2)\Big|_{(k,n,\ell)\,\text{coprime}} = (-)^\ell \sum_{J_R, BPS\ states} 2(J_R^3)^2(-)^{2J_R^3}\,. \tag{2.8}$$

[1] A free sigma model on $R^4 \times T^4$ is factored out here, and our definition differs by a factor of 2 from [2].

When they are not coprime, the black hole can fragment, and the situation is more complicated due to multiple contributions in $\mathcal{E}_2$ [2]. In this paper we will always avoid this complication by choosing coprime charges.

We should note that $Z(q,y)$ is also the modified elliptic genus of T^4, i.e.

$$\mathcal{E}_2^{(1)} = \sum_{n,\ell} \widehat{c}(n,\ell) q^n y^\ell = Z(q,y). \tag{2.9}$$

This corresponds to the coprime D1-D5 system with $k = 1 = Q_1 = Q_5$. By writing

$$Z(q,y) = \sum_m \widehat{c}(4m) q^m \sum_k q^{k^2} y^{2k} + \sum_m \widehat{c}(4m-1) q^m \sum_k q^{k^2+k} y^{2k+1} \tag{2.10}$$

and using (2.4) along with the standard Fourier expansion of the theta function

$$\vartheta_1(y|q) = i \sum_{n\in\mathbb{Z}} (-1)^n q^{(n-1/2)^2/2} y^{n-1/2} \tag{2.11}$$

one can reorganize the generating functions for $\widehat{c}$ as

$$\begin{aligned} \sum_m \widehat{c}(4m) q^m &= -q^{\frac{1}{4}} \eta(q)^{-6} \sum_{m\in\mathbb{Z}} q^{m^2+m}, \\ \sum_m \widehat{c}(4m-1) q^m &= q^{\frac{1}{4}} \eta(q)^{-6} \sum_{m\in\mathbb{Z}} q^{m^2}. \end{aligned} \tag{2.12}$$

These expressions will analyzed microscopically below in section 4.

3. The 4D modified elliptic genus

In this section we use the conjecture of [1] to transform the 5D degeneracies into 4D ones. The fact that the $\widehat{c}$ coefficients depend only on the combination $4nk - \ell^2$ is very encouraging, for the following reason. We expect the 1/8 BPS 5D degeneracies to be related to degeneracies of 1/8 BPS black holes in 4D, and in 4D U-duality implies [9] that the black hole entropy must depend on the unique quartic invariant of $E_{7,7}$, the so-called Cremmer-Julia invariant [10]. In an $\mathcal{N} = 4$ language, this invariant takes the form

$$\mathcal{J} = q_e^2 q_m^2 - (q_e \cdot q_m)^2 \tag{3.1}$$

where q_e and q_m are the electric and magnetic charge vectors for $\mathcal{N} = 4$ BPS states. (See e.g. [5] for details on the notation.) This is precisely the dependence of $\widehat{c}$ on n, m, ℓ, provided we identify

$$k = \frac{1}{2} q_e^2, \qquad n = \frac{1}{2} q_m^2, \qquad \ell = q_e \cdot q_m. \tag{3.2}$$

Note that from the purely 5D point of view, there was no obvious reason that $\widehat{c}$ should depend only on the combination $4nk - \ell^2$ as there is no 5D U-duality which mixes spins with charges.

Let us now derive the identification (3.2) from the dictionary of [1], beginning from the IIB spinning 5D D1-D5-n black hole of the previous section. First we T-dual on S^1 to obtain a black hole with spin $\frac{\ell}{2}$, F-string winding n, Q_1 D0-branes, and Q_5 D4-branes. Now

JHEP06(2006)037

T-dual so that there are Q_1+Q_5 D2 branes with intersection number $Q_1 Q_5 = k$ on the T^4. Next we compactify on a single center Taub-NUT, whose asymptotic circle we identify as the the new M-theory circle. The result is three orthogonal sets of (n, Q_1, Q_5) D2-branes on T^6, ℓ D0-branes, and one D6-brane. For IIA D-brane configurations with D0, D2, D4, D6 charges $(q_0, q_{ij}, p^{ij}, p^0)$, where $i = 1, ...6$ runs over the T^6 cycle and $p^{ij} = -p^{ji}$, $q_{ij} = -q_{ji}$ $\mathcal{J}$ reduces to[2]

$$\mathcal{J} = \frac{1}{12}(q_0 \epsilon_{ijklmn} p^{ij} p^{kl} p^{mn} + p^0 \epsilon^{ijklmn} q_{ij} q_{kl} q_{mn}) \\ -p^{ij} q_{jk} p^{kl} q_{li} + \frac{1}{4} p^{ij} q_{ij} p^{kl} q_{kl} - (p^0 q_0)^2 + \frac{1}{2} p^0 q_0 p^{ij} q_{ij}. \tag{3.3}$$

For our D0-D2-D6 configuration, we can pick a basis of cycles without loss of generality such that the nonzero charges are

$$p^0 = 0, \quad q_0 = \ell, \quad q_{12} = -q_{21} = n, \quad q_{34} = -q_{43} = Q_1, \quad q_{56} = -q_{65} = Q_5 \tag{3.4}$$

Then (3.3) reduces to

$$\mathcal{J} = 4nk - \ell^2, \tag{3.5}$$

which, as stated above, is exactly the argument of (2.3).

According to [1] the weighted degeneracy of the 4D black hole resulting from U-duality and Taub-NUT compactification equals that of the original 5D black hole, when J_R^3 in (2.8) is identified with the generator J^3 of $\mathbb{R}^3$ rotations in 4D. Note that, since $\mathcal{J}$ is odd if and only if ℓ is, we may trade $(-)^\ell$ for $(-)^\mathcal{J}$ in (2.8). Therefore, for fixed coprime charges, the weighted 4D BPS degeneracy depends only on the the Cremmer-Julia invariant and is given by

$$\sum_{J^3, BPS\ states} 2(J^3)^2 (-)^{2J^3} = (-)^\mathcal{J} \widehat{c}(\mathcal{J}). \tag{3.6}$$

Note that, although this formula for the 4D BPS degeneracy was derived assuming a specific D6-D2-D0 configuration, it applies to all D-brane configurations by U-duality.

As a first check on this conjecture, we note that for large charges $\widehat{c}(\mathcal{J}) \sim e^{\pi\sqrt{\mathcal{J}}}$. From the supergravity solutions Area $= 4\pi\sqrt{\mathcal{J}}$, so there is agreement with the Bekenstein-Hawking entropy.

As an example, let's consider the modified elliptic genus for the D4-D0 black hole on T^6, in which we fix the D4 charges and sum over D0 charge q_0. Consider the T^6 of the form $T^2 \times T^2 \times T^2$ with $\alpha_1, \alpha_2, \alpha_3$ being the three 2-cycles associated with the T^2's. Let A^1, A^2, A^3 be the dual 4-cycles. We shall consider the D4-brane wrapped on the cycle $[P] = A^1 + A^2 + A^3$. Its triple self-intersection number is $D = P \cdot P \cdot P = 6$. From (3.3) we have

$$\mathcal{J} = 4q_0. \tag{3.7}$$

We then have

$$\mathcal{E}_2(q) = \sum_{q_0 \in \mathbb{Z}} \widehat{c}(4q_0) q^{q_0} = -q^{\frac{1}{4}} \eta(q)^{-6} \sum_{m \in \mathbb{Z}} q^{m^2+m}. \tag{3.8}$$

according to (2.12).

[2] See e.g. [11], equation (66), and take $p^0 = p_{87}$, $p_{8i} = 0$, etc. Our definition of $\mathcal{J}$ differs from that of [11] by a sign.

A straightforward generalization of this example is the D4-D2-D0 system, where we wrap (q_1, q_2, q_3) D2 branes on the 2-cycles $(\alpha_1, \alpha_2, \alpha_3)$. In this case, the Cremmer-Julia invariant becomes

$$\mathcal{J} = 4(q_0 + q_1 q_2 + q_1 q_3 + q_2 q_3) - (q_1 + q_2 + q_3)^2 \tag{3.9}$$

and the sum over q_0 produces

$$\mathcal{E}_2(q) = \sum_{q_0 \in \mathbb{Z}} (-1)^{\mathcal{J}} \widehat{c}(\mathcal{J}) q^{q_0} = \begin{cases} -q^{\frac{1}{4}} \eta(q)^{-6} \sum_{m\in\mathbb{Z}} q^{m^2+m-\frac{1}{4}\tilde{\mathcal{J}}} & q_1 + q_2 + q_3 \text{even} \\ -q^{\frac{1}{4}} \eta(q)^{-6} \sum_{m\in\mathbb{Z}} q^{m^2-\frac{1}{4}\tilde{\mathcal{J}}-\frac{1}{4}} & q_1 + q_2 + q_3 \text{odd} \end{cases} \tag{3.10}$$

where $\tilde{\mathcal{J}} = 4(q_1 q_2 + q_1 q_3 + q_2 q_3) - (q_1 + q_2 + q_3)^2$. Now let us turn to the 4D derivation of (3.8) and (3.10).

4. Microscopic derivation in 4D

In this section we sketch a derivation of (3.8) and (3.10) using a 4D microscopic analysis. The derivation is not complete because, as we will discuss below, we ignore some potential subtleties associated to the fact that P is not simply connected. In principle it should be possible to close this gap. A microscopic description of T^6 black holes using the M-theory picture of wrapped fivebranes has been given in [8] , adapting the description given in [12] for a general Calabi-Yau, in terms of a $(0, 4)$ 2D CFT living on the M-theory circle. For uniformity and simplicity of presentation, we here will use the IIA description in which fivebrane momenta around the M-theory circle become bound states of D0 branes to D4 branes.

As above (3.7) we examine the special case of the D4-D0 system wrapped on $[P] = A^1 + A^2 + A^3$. The D4-D0 system can be described in terms of the quantum mechanics of q_0 D0-branes living on the D4-brane world volume P. The D4-brane world volume P is holomorphically embedded in the T^6. One can compute its Euler character, $\chi(P) = 6$. It follows from the Riemann-Roch formula that the only modulus of P is the overall translation in T^6.[3] Since $\chi(P) = 6$, P has $4 + 2b_1$ 2-cycles. By the Lefschetz hyperplane theorem we have $b_1(P) = b_1(T^6) = 6$, and therefore $b_2(P) = 16$. All but one of the 2-cycles come from the intersection of P with $\binom{6}{4} = 15$ 4-cycles in T^6. We will be mostly interested in 3 of these, denoted by $\tilde{\alpha}_i$, corresponding to intersections of A^i with P. Turning on fluxes along these three 2-cycles corresponds to having charges of D2-branes wrapped on the α_i's. Their intersection numbers are

$$\tilde{\alpha}_i \cdot \tilde{\alpha}_j = \begin{cases} 0, & i = j \\ 1, & i \neq j \end{cases} \tag{4.1}$$

There is, however, one extra 2-cycle in P, which we shall denote by β, that does not correspond to any cycle in the T^6.

[3] The dual line bundle $\mathcal{L}_P$ of the divisor P has only one holomorphic section. However as T^6 is not simply connected, the line bundle $\mathcal{L}_P$ is not only determined by $c_1(\mathcal{L}_P) = [P]$. In fact the translation of T^6 takes it to a different line bundle.

One can show from the adjunction formula that $c_1(P)$ is Poincaré dual to $-(\tilde{\alpha}_1+\tilde{\alpha}_2+\tilde{\alpha}_3)$. It then follows from Hirzebruch signature theorem that

$$\sigma(P) = -\frac{2}{3}\chi(P) + \frac{1}{3}\int_P c_1^2 = -2. \tag{4.2}$$

We conclude that the intersection form on P is odd (and that P is not a spin manifold). Essentially the unique way to extend (4.1) to an odd rank 4 unimodular quadratic form is to have an extra 2-cycle γ with

$$\gamma \cdot \tilde{\alpha}_i = 1, \qquad \gamma \cdot \gamma = 1. \tag{4.3}$$

Now if we choose $\beta = 2\gamma - \sum \tilde{\alpha}_i$, we have

$$\beta \cdot \tilde{\alpha}_i = 0, \qquad \beta \cdot \beta = -2. \tag{4.4}$$

Note that $(\tilde{\alpha}_i, \beta)$ is not an integral basis for $H_2(P,\mathbb{Z})$, yet β is the smallest 2-cycle that doesn't intersect $\tilde{\alpha}_i$. The total intersection form on P is the sum of this rank 4 form together with 6 copies of σ_1 coming from the 12 other 2-cycles in P.

Now one can turn on gauge field flux on the D4-brane world volume along β, which does not correspond to any D2-brane charge. This flux nevertheless induces D0-brane charge. There is a subtlety in the quantization of this flux. As well known, the curvature of the D4-brane world volume induces an anomalous D0-brane charge $-\chi(P)/24 = -\frac{1}{4}$. In order that the total D0 charge be integral the flux along the cycle β on the D4-brane must be half-integer, i.e. of the form $(m+\frac{1}{2})\beta$. The total induced D0-brane charge is $\Delta q_0 = -\frac{1}{2}(m+\frac{1}{2})^2\beta\cdot\beta - \frac{1}{4} = m^2+m$, which is indeed an integer.[4]

We ignore here the facts arising form nonzero $b_1(P)$ that there is a moduli space of flat connections as well as overall T^6 translations which must be quantized. These factors are treated in the language of the 2D CFT in [8]. They are found to lead to extra degrees of freedom which are however eliminated by extra gauge constraints. A complete microscopic derivation, not given here, would have to show that a careful accounting of these factors give a trivial correction to our result.

It is now straightforward to reproduce (3.8). Each D0-D4 bound state is in a hypermultiplet which contributes minus one to $Tr[2(J^3)^2(-)^{2J^3}]$. Counting the number of ways of distributing n D0-branes among the $\chi(P)=6$ ground states of the supersymmetric quantum mechanics, and then summing over n, gives the factor of $q^{1/4}\eta(q)^{-6} = \prod_{k=1}^{\infty}(1-q^k)^{-6}$ in (3.8). Including finally the sum over fluxes on β, we precisely reproduce the degeneracy (3.8)!

Let us now consider the more general case of D4-D2-D0 system. Again we shall assume $(p^1,p^2,p^3)=(1,1,1)$. The D2-brane charges are labelled by (q_1,q_2,q_3). The bound state is described by the D4-brane with D2-brane dissolved in its world volume. We end up with the gauge flux

$$F = (m+1/2)\beta + \sum_{i=1}^{3} q_i\delta_i, \qquad \delta_i \cdot \tilde{\alpha}_j = \delta_{ij}. \tag{4.5}$$

[4] In the M-theory picture the anomalous D0 charge is the left-moving zero point energy $-\frac{c_L}{24} = -\frac{1}{4}$, and the 2-cycle fluxes correspond to momentum zero modes of scalars on a Narain lattice.

In above expression δ_i is defined up to a shift of an integer multiple of β. Since we are summing over m, this ambiguity is irrelevant. We can choose $\delta_i = \gamma - \tilde{\alpha}_i$. The total induced D0-brane charge is then

$$\begin{aligned}\Delta q_0 &= -\int \frac{1}{2}F^2 - \frac{1}{4} \\ &= \left(m+\frac{1}{2}\right)^2 + \left(m+\frac{1}{2}\right)\sum q_i + \frac{1}{2}\sum q_i^2 - \frac{1}{4} \\ &= \left(m+\frac{1}{2}+\frac{1}{2}\sum q_i\right)^2 - \frac{1}{12}D^{AB}q_A q_B - \frac{1}{4},\end{aligned} \tag{4.6}$$

where D^{AB} is the inverse matrix of $D_{AB} \equiv D_{ABC}p^C$,

$$D^{AB}q_A q_B = 3(2q_1q_2 + 2q_2q_3 + 2q_3q_1 - q_1^2 - q_2^2 - q_3^2). \tag{4.7}$$

Note that $\frac{1}{3}D^{AB}q_Aq_B = 0 \bmod 4$ if $\sum q_i$ is even, and $\frac{1}{3}D^{AB}q_Aq_B = -1 \bmod 4$ if $\sum q_i$ is odd. Therefore Δq_0 is always an integer, as expected. The Cremmer-Julia invariant is in this case

$$\mathcal{J} = 4\left(q_0 + \frac{1}{12}D^{AB}q_Aq_B\right). \tag{4.8}$$

The counting of D0-brane states as before gives the generating function

$$\sum_{q_0}(-)^{\mathcal{J}}c(\mathcal{J})q^{q_0} = -\prod_{k=1}^{\infty}(1-q^k)^{-6}\sum_{m\in\mathbb{Z}} q^{m^2+m-\frac{1}{12}D^{AB}q_Aq_B} \tag{4.9}$$

in the case $\sum q_i \in 2\mathbb{Z}$ and $\mathcal{J} \equiv 0 \bmod 4$, and

$$\sum_{q_0}(-)^{\mathcal{J}}c(\mathcal{J})q^{q_0} = -\prod_{k=1}^{\infty}(1-q^k)^{-6}\sum_{m\in\mathbb{Z}} q^{m^2-\frac{1}{12}D^{AB}q_Aq_B-\frac{1}{4}} \tag{4.10}$$

in the case $\sum q_i \in 2\mathbb{Z}+1$ and $\mathcal{J} \equiv -1 \bmod 4$. These are precisely the degeneracies (3.10) we derived from 5D earlier!

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JHEP06(2006)037